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Price Indices and Nonlinear Mean-Reversion of Real Exchange Rates

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The purpose of this article is to apply a symmetric band–threshold autoregressive model to investigate several interesting issues regarding purchasing power parity (PPP). We find that the nonlinear adjustment toward PPP is sensitive to price indices and is supported if a traded-goods real exchange rate is applied. Moreover, we also uncover the sources of the real exchange rate adjustments toward PPP. Finally, our evidence points out that the estimated half-life with a large shock, based on a generalized impulse response function, can be explained by nominal rigidities.

JEL Classification: F31

1. Introduction

Are price indices crucial for the existence of a nonlinear mean reversion of real exchange rates? Do prices or exchange-rate adjustments dominate when deviations from purchasing power parity (PPP) occur? Is the half-life implied by a nonlinear model reasonable? The purpose of this article is to address the above three questions for the UK and New Zealand over the period of recent float.

The PPP hypothesis has been one of the most intensive research issues in empirical international finance over the past two decades. The rationale behind it is a simple arbitrage hypothesis, which results in a linear adjustment of deviations from PPP and the stationarity of real exchange rates. Empirically, existing evidence based on unit-root tests provide mixed results for PPP (Abuaf and Jorion 1990; Mark 1990; O'Connell 1998a).

Theoretically there are several reasons for the nonlinear adjustment of deviations from PPP, such as the existence of market frictions or transaction costs (Sercu, Uppal, and Van Hulle 1995). In addition, models of pricing to market and exchange rate pass-through give rise to impediments to goods' arbitrage (Krugman 1987; Froot and Klemperer 1989). The implication is that the speed of adjustment of deviations from PPP depends on the magnitude of the deviations.

On the other hand, the adoption of a price index is crucial in examining PPP. Several authors have argued that the consumer and producer price indices (CPI and PPI, respectively)

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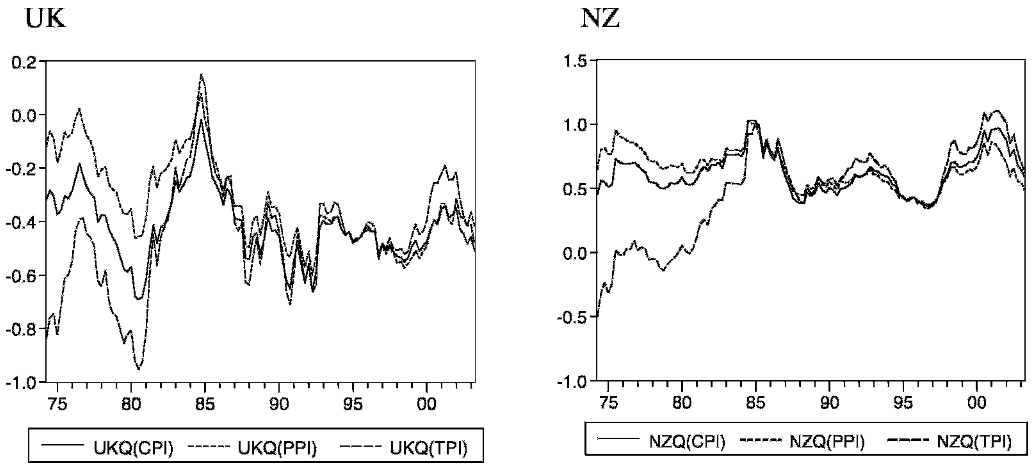

Figure 1. Plots of Real Exchange Rates for the UK and New Zealand

do not correspond to their theoretical counterparts and contain measurement errors and aggregation biases (Cheung and Lai 1993; Imbs et al. 2005). In addition, the commodity basket for CPI and PPI includes nontradable goods, which impart a nonstationary component to real CPI or PPI exchange rates (Engel 1999). Recently, Xu (2003) argues that the price index of traded goods (TPI) is the appropriate one for PPP since it reflects the behavior of arbitrage better than the CPI or PPI.

The purpose of this article, therefore, is to examine the nonlinear dynamics of TPI-based real exchange rates. Several authors have applied a smooth transition autoregressive (STAR) model to capture the nonlinear dynamics of real exchange rates, as it allows for a smooth adjustment between regimes (Michael, Nobay, and Peel 1997; Taylor, Peel, and Sarno 2001). There are several reasons for us to apply a band-threshold autoregressive (TAR) instead of a STAR-type model in our empirical analysis. First, our empirical evidence fails to reject the unit-root hypothesis against the hypothesis of a nonlinear STAR stationary process based on the test provided by Kapetanios, Shin, and Snell (2003).[1] Second, the plots of CPI-, PPI-, and TPI-based real exchange rates in Figure 1 show that the TPI-based real rate has the largest variation among these three real rates. Third, our empirical evidence rejects the null hypothesis of linearity against TAR-type nonlinearity and supports the hypothesis that real exchange rates are TAR-type stationary.

Several empirical studies have applied a threshold-type process to examine the nonlinear mean-reversion of real exchange rates (Obstfeld and Taylor 1997; O'Connell 1998b; Taylor 2001; Sarno, Taylor, and Chowdhury 2004). There are several restrictions embedded in existing literature. First, the threshold value is not estimated based on an algorithm (O'Connell 1998b). Second, the lag order of models or that of the threshold variable is not selected appropriately (Obstfeld and Taylor 1997; Taylor 2001). Third, the symmetric assumption of a band-TAR

[1] Our finding from the CPI-based real rate is different from that in Kapetanios, Shin, and Snell (2003), which may be due to the different empirical periods adopted. The empirical period in Kapetanios, Shin, and Snell (2003) covers both fixed and floating regimes, but the period in our article covers only the recent float.

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model is not examined empirically (Sarno, Taylor, and Chowdhury 2004). Fourth, none of the previously mentioned articles, except for that of Sarno, Taylor, and Chowdhury (2004), provides a linearity test to support the appropriateness of a TAR specification.

If deviations from PPP are observed, then it is interesting to ask whether the reversion toward parity is attributed to the adjustment of nominal exchange rates or price levels. For this purpose, we apply the threshold vector-error-correction model (TVECM) provided by Hansen and Seo (2002) and Seo (2003) to examine the sources of adjustment toward PPP. Apart from unveiling the sources of adjustment toward PPP, we are also interested in the speed of parity reversion. If the failure of PPP were to be attributed to stickiness in nominal prices, then presumably we would expect substantial convergence to PPP over one to two years (Rogoff 1996). The generalized nonlinear impulse response functions introduced by Koop, Pesaran, and Potter (1996) are adopted to assess whether the half-life from a nonlinear model helps to resolve the PPP puzzle of the slow speed of adjustment of real exchange rates asserted by Rogoff (1996).

Our empirical findings point out that the nonlinear dynamics in deviations from PPP are supported by TPI-based, but not PPI-based or CPI-based, real exchange rates. Moreover, the adjustments toward PPP are attributed to the nominal exchange rate and domestic price adjustments. Finally, we find that the speed of adjustment of the deviations from PPP depends on the magnitude of the deviations. The estimated half-life relative to the band edge of equilibrium is less than four quarters with a large shock, which is consistent with the explanation based on price stickiness.

The remainder of this article is organized as follows. Section 2 describes the econometric methodology. We provide our empirical results in section 3. Section 4 provides our conclusions.

2. The Econometric Model and its Estimation Methods

Following Balke and Fomby (1997), the band-TAR model for the demeaned real exchange rate can be written as follows:

$$\begin{aligned} q_t = &\left(\kappa\left(1-\sum_{i=1}^{m}\alpha_i\right)+\sum_{i=1}^{m}\alpha_i q_{t-i}\right)1(q_{t-d}\le -\kappa)+\left(\sum_{i=1}^{m}\beta_i q_{t-i}\right)1(|q_{t-d}|<\kappa) \\ &+\left(\kappa\left(1-\sum_{i=1}^{m}\gamma_i\right)+\sum_{i=1}^{m}\gamma_i q_{t-i}\right)1(q_{t-d}\ge\kappa)+\varepsilon_t, \end{aligned} \tag{1}$$

where q_t is the demeaned real exchange rate and q_{t-d} is the threshold variable with d chosen among $1,2\ldots m$;$1(q_{t-d} \le -\kappa)$, $1(|q_{t-d}| < \kappa)$, and $1(q_{t-d} \ge \kappa)$ are indicator variables that take the value of 1 when the inequality in the parentheses is satisfied, and the value of 0 otherwise. The error term ε_t is identically, independently, and normally distributed with a zero mean and a constant variance of σ^2.

The model allows for the band of inaction, $[-\kappa, \kappa]$, since profits from commodity arbitrage are small compared to transaction costs within the band. If $\sum\beta_i = 1$, $\sum\gamma_i < 1$, and $\sum\alpha_i < 1$, $i = 1,\ldots,m$, then there is no tendency for q_t to be mean reverting within the band, but q_t does exhibit a tendency to revert back to the edge of the band when it lies outside of the band. The model in Equation 1 also allows for symmetric adjustments of the real exchange rate (the case where $\alpha_i = \gamma_i$ for all i) when it lies outside of the band. The convergence speed, relative to the band edge, to the

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equilibrium is $1 - \sum \alpha_i$.[2] Hansen (1997, 1999) suggests estimating the AR parameters as well as a two-dimensional grid search over (κ, d) by applying sequential conditional least squares.[3]

To justify a parsimonious specification of our model, we examine the following hypothesis sequentially.

$$\mathrm{H}_0^{\mathrm{A}} : \alpha_1 = \gamma_1, \cdots, \text{and } \alpha_m = \gamma_m,$$

$$\mathrm{H}_0^{\mathrm{B}} : \kappa = 0 |\ \mathrm{H}_0^{\mathrm{A}},$$

$$\mathrm{H}_0^{\mathrm{C(1)}} : \beta_1 + \ldots + \beta_m = 1 |\ \mathrm{H}_0^{\mathrm{A}}, \text{ and}$$

$$\mathrm{H}_0^{\mathrm{C(2)}} : \alpha_1 + \ldots + \alpha_m = \beta_1 + \ldots + \beta_m = 1 |\ \mathrm{H}_0^{\mathrm{A}}.$$

The hypothesis $\mathrm{H}_0^{\mathrm{A}}$ is the symmetric hypothesis, which claims that the arbitrage forces are identical regardless of whether the deviations from PPP are above or below the arbitrage band. If $\mathrm{H}_0^{\mathrm{A}}$ is not rejected, then we impose the symmetric assumption and then test for linearity ($\mathrm{H}_0^{\mathrm{B}}$) to examine whether a symmetric band-TAR model is appropriate for describing the dynamics of real exchange rates. If $\mathrm{H}_0^{\mathrm{B}}$ is rejected, then we test the hypotheses $\mathrm{H}_0^{\mathrm{C(1)}}$ and $\mathrm{H}_0^{\mathrm{C(2)}}$, respectively. If $\mathrm{H}_0^{\mathrm{C(1)}}$ fails to be rejected but $\mathrm{H}_0^{\mathrm{C(2)}}$ is rejected, this implies that the real exchange rate follows an I(1) process within the band but a stationary autoregressive process outside of the band. In this case, the real exchange rate is considered stationary overall although it has different time-series properties in different regimes and although its adjustment is nonlinear.

A likelihood ratio statistic is applied to examine the above-mentioned hypotheses. Since threshold κ is not identified under $\mathrm{H}_0^{\mathrm{B}}$, the asymptotic distribution of the likelihood ratio statistic is nonstandard. We simulate the marginal significance levels of the likelihood ratio (LR) statistic by constructing a parametric bootstrap procedure suggested by Hansen (1997, 1999).

Once the empirical evidence supports the overall stationarity of the real exchange rate with symmetric adjustments outside of the band, we then analyze the sources of adjustments to PPP. The appropriateness of a symmetric band-TAR model is justified in our empirical section. A nonlinear error-correction model with a symmetric speed of adjustment is adopted to examine whether nominal exchange rates or price levels are responsible for the deviations from PPP during the adjustment process. The nonlinear threshold error-correction model is described as follows:

$$\begin{aligned}
\Delta x_t = & \left(-\lambda_0^x + \lambda_1^x q_{t-1} + \sum_{i=1}^{m} \phi_i^x \Delta y_{t-i} \right) 1(q_{t-1} \leq -\tau) \\
& + \left(\rho_1^x q_{t-1} + \sum_{i=1}^{m} \psi_i^x \Delta y_{t-i} \right) 1(|q_{t-1}| < \tau) \qquad \text{for } x = s, p^*, p, \qquad (2) \\
& + \left(\lambda_0^x + \lambda_1^x q_{t-1} + \sum_{i=1}^{m} \phi_i^x \Delta y_{t-i} \right) 1(q_{t-1} \geq \tau) + u_t,
\end{aligned}$$

[2] The real exchange rate is in equilibrium throughout the entire interval $[-\kappa, \kappa]$ of the band. It still makes sense to speak of convergence to equilibrium at the speed $1 - \sum \alpha_i$, where this is interpreted as convergence relative to the band.

[3] Under the assumption of normality, the least-squares estimators are equivalent to maximum likelihood estimators. The delay and threshold parameters are chosen simultaneously by minimizing the residual variance.

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where $y_t = \left(s_t, p_t^*, p_t\right)'$, $\phi_i^x = \left(\phi_{1i}^x, \phi_{2i}^x, \phi_{3i}^x\right)$, and $\psi_i^x = \left(\psi_{1i}^x, \psi_{2i}^x, \psi_{3i}^x\right)$. The statistical significance of the error-correction parameters, $\lambda_1 = \left(\lambda_1^s, \lambda_1^p, \lambda_1^{p*}\right)'$ and $\rho_1 = \left(\rho_1^s, \rho_1^p, \rho_1^{p*}\right)'$ are supposed to give information about which variables among s_t, p_t^*, and p_t dominate in each regime.

If $\mathrm{H}_0^{C(1)}$ is not rejected, then the error-correction term in the middle regime of a threshold VECM should have no effect on any of the variables, so that ρ_1 will be a zero vector. Hansen and Seo (2002) and Seo (2003) suggest using a grid search over a two-dimensional space consisting of a cointegrating vector and the threshold value. If real exchange rates are stationary, then the cointegrating vector is assumed fixed and is equal to (1, 1, −1). Furthermore, we predetermine the threshold variable to the error-correction term q_{t-1} so that the grid search applies to the threshold value only. The log-likelihood function proposed by Hansen and Seo (2002) and Seo (2003) is applied to estimate the parameters of interest.[4]

One way to obtain further insights into the mean-reverting properties of the estimated nonlinear model is to calculate the half-lives of the real exchange rates. The conventional measure of half-life is biased when real exchange rates follow a nonlinear TAR process (Taylor 2001). In a nonlinear framework, we evaluate the propagation mechanism of shocks to the deviation from PPP by constructing a generalized impulse response function (GIRF). Following Koop, Pesaran, and Potter (1996), the GIRF is defined as the difference between two conditional expectations:

$$\mathrm{GIRF}_q(\eta, v_t, \omega_{t-1}) = E\left[q_{t+\eta} \middle| v_t, \omega_{t-1}\right] - E\left[q_{t+\eta} \middle| \omega_{t-1}\right],$$

where $\mathrm{GIRF}_{q.}$ is the GIRF of the real exchange rate; η is the forecasting horizon; v_t is the shock to the process at time t; ω_{t-1} is the history of the variable, which is the set of the historical data of q_t, as suggested by Koop, Pesaran, and Potter (1996); and E[·] is the conditional expectation operator. With nonlinear models, the GIRF is characterized by shock and sign asymmetry.

3. Empirical Investigation

Data Description

The empirical period starts with 1974:2 and ends with 2003:2. The reason for starting the empirical period with 1974:2 is to remove transition periods after adopting a flexible exchange rate system in 1973:2. Variables for the United Kingdom (UK), New Zealand (NZ), and the United States (US) are obtained from the International Monetary Fund's International Financial Statistics (IFS) database.[5] They include the nominal exchange rate of the pound and New Zealand dollar, the consumer and producer price indices, and export and import price indices for all countries. The TPI is a weighted average of the export price index (line 74) and import price index (line 75), with the weights composed of the shares of total exports (line 70)

[4] A restriction of this algorithm is the lack of a theory of inference. We, therefore, report the conventional standard deviations for these estimated parameters.

[5] The reason for us to examine these three countries is that empirical results from other major industrial countries fail to support our findings in Tables 1 and 2.

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and total imports (line 71) in total trade, respectively. The exchange rate is the amount of domestic currency per U.S. dollar.

Empirical Results

The stationarity of real exchange rates is required for us to model them using nonlinear processes. We apply the unit-root tests provided by Ng and Perron (2001) to examine the stationarity of real exchange rates, in which the lag length is selected by the rule of modified Akaike information criterion (AIC). Our findings fail to reject the unit-root hypothesis of real exchange rates, regardless of the price indices (results are not reported but are available upon request from the authors). The conventional linear unit-root tests are shown to lack power if real exchange rates follow a nonlinear threshold process (Taylor 2001). Therefore, the failure to reject the unit-root hypothesis with conventional linear unit-root tests could be due to the fact that the variables under investigation are nonlinear.

Two different types of nonlinear models have been widely applied in the literature, one being a STAR-type model and the other being a TAR-type model. We apply the nonlinear unit-root test provided by Kapetanios, Shin, and Snell (2003) to examine the unit-root hypothesis against the nonlinear STAR stationary process.[6] Our findings fail to reject the unit-root hypothesis of real exchange rates except for the CPI-based real exchange rate in New Zealand (results are not reported but are available upon request from the authors). Hence, a STAR model may not be appropriate to examine the dynamics of real exchange rates. We apply a band-TAR model for our empirical purposes. The appropriateness of a symmetric band-TAR model and the nonlinear band-TAR stationarity of TPI-based real exchange rates is examined and supported in the following sections.

To determine the lag length of the band-TAR model, we follow the conventional strategy to start from a linear AR(1) and then apply the Ljung-Box Q-test to check the whiteness of the estimated residuals (Enders and Siklos 2001). If the residuals are nonwhite, we then increase the lag order by one until they are whitened. After determining the appropriate lag length, we estimate Equation 1 to obtain the unrestricted slope coefficients in each regime. The grid search ranges from the 70th to the 90th percentile of the arranged sample for the threshold value. The LR statistic is then applied to test for the null hypothesis H_0^A against the asymmetric band-TAR model. The LR statistics for the UK's CPI-, PPI-, and TPI-based rates on the upper panel of Table 1 fail to reject the null hypothesis of H_0^A at the 10% level of significance, since their corresponding *p* values are 0.370, 0.717, and 0.870, respectively. Similar findings are obtained for New Zealand.[7]

We impose the symmetric assumption and then test the appropriateness of a symmetric band-TAR specification by testing the hypothesis of $\kappa = 0$. The findings from the lower panel of Table 1 indicate that the hypothesis of $\kappa = 0$ is rejected, at the 10% level of significance, only for TPI-based real exchange rates in both countries.[8] There is no significant evidence to support

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[6] The lag order in the nonlinear unit-root test is set to be the same as that in the Ng-Perron test.

[7] One may ask to what extent the inferences from the symmetry test are affected if real exchange rates are nonstationary. We, therefore, simulate the finite sample distribution of the likelihood ratio statistic for symmetry assuming nonstationary real exchange rates. The *p* values of the symmetric test for UK's (NZ's) CPI-, PPI-, and TPI-based real exchange rates are 0.299 (0.695), 0.619 (0.783), and 0.872 (0.992), respectively. Our conclusions from Table 1 are not affected by this change.

[8] We also simulate the finite sample distribution of the likelihood ratio statistic for linearity assuming nonstationary real exchange rates. The *p* values of the linear test for UK's (NZ's) CPI-, PPI-, and TPI-based real exchange rates are 0.270 (0.145), 0.185 (0.167), and 0.057 (0.097), respectively. Therefore, our conclusions about linearity from Table 1 are not affected even though real exchange rates are nonstationary.

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Table 1. Symmetric and Linear Tests

	UK			NZ		
	CPI	PPI	TPI	CPI	PPI	TPI
$H_0^A: \alpha_1 = \gamma_1, \ldots,$	12.741	8.044	4.216	5.507	1.369	0.221
and $\alpha_m = \gamma_m$	[0.370]	[0.717]	[0.870]	[0.687]	[0.784]	[0.996]
$H_0^B: \kappa = 0$ (Linear AR)	13.083	14.570	18.813*	12.010	4.902	10.121**
$H_A^B: \kappa > 0$ (Symmetric band-TAR)	[0.285]	[0.195]	[0.045]	[0.171]	[0.200]	[0.073]

UK and NZ indicate the United Kingdom and New Zealand, respectively. The numbers in the table are likelihood ratio statistics. Figures in square brackets are the marginal significance levels generated by the bootstrapping method described in the text.
* Significance at the 5% level.
** Significance at the 10% level.

the nonlinear adjustment in the cases of the PPI- and CPI-based real exchange rates. This finding is interesting and may be explained by the fact that the calculation of both the CPI and PPI involves nontradable goods and, hence, they are poor proxies for tradable prices that characterize real exchange rates. Hence, our findings point out that the adjustment in terms of deviations from PPP is sensitive to price indices. Since the nonlinear adjustments of TPI-based real exchange rates are supported empirically, we estimate a symmetric band-TAR model and report our empirical results in Table 2.

In Table 2, L, M, and U indicate the lower, middle, and upper regimes corresponding to the regime with $q_{t-d} \leq -\kappa$, $|q_{t-d}| < \kappa$, and $q_{t-d} \geq \kappa$, respectively. The degree of mean reversion in different regimes is measured by the sum of the estimated autoregressive parameters, which is 0.75 (0.71) in the outer regimes (the lower and upper regimes) and 0.82 (0.94) in the middle

Table 2. Results for Symmetric Band-TAR Model (TPI)

	UK		NZ	
	L and U	M	L and U	M
θ_1	1.64 (0.17)	1.00 (0.08)	0.64 (0.09)	1.20 (0.11)
θ_2	−1.08 (0.31)	−0.38 (0.15)	0.07 (0.11)	−0.26 (0.10)
θ_3	0.47 (0.28)	0.52 (0.15)	—	—
θ_4	−0.28 (0.18)	−0.32 (0.12)	—	—
$\sum \theta_i$	0.75	0.82	0.71	0.94
d	2		2	
κ	0.236		0.445	
LR1	6.630 [0.112]		2.218 [0.418]	
LR2	13.517** [0.095]		12.310** [0.082]	
Q (16)	20.37 [0.20]		22.72 [0.12]	
Q^2(16)	22.67 [0.12]		21.26 [0.17]	

UK and NZ indicate the United Kingdom and New Zealand, respectively. L, M, and U represent the regimes that are defined as $q_{t-d} \leq -\kappa$, $|q_{t-d}| < \kappa$, and $q_{t-d} \geq \kappa$, respectively. Here, θ_i is the same as α_i, $i = 1,\ldots m$, in Equation 1 when regimes L and U are applied, but θ_i denotes β_i, $i = 1\ldots m$, if regime M is applied. Figures in parentheses denote the estimated standard errors, and d is the lag order of the threshold variable. The test statistic LRi is the likelihood ratio statistic for the hypothesis of $H_0^{C(i)}$. Figures in square brackets represent the marginal significance levels generated by the bootstrapping method described in the text. Q(j) and Q^2(j) denote the Ljung-Box autocorrelation test statistics for up to jth-order autocorrelation for estimated residuals and squared residuals, respectively, which have χ^2 distributions with j degrees of freedom. The marginal significance levels of the Ljung-Box statistics are given in square brackets. The '—' symbol indicates that an estimate is not computed.
** Significance at the 10% level.

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regime for the case of the UK (New Zealand). Obviously, the mean-reverting adjustment in the outer regimes is less persistent than that in the middle regime. The nonlinear least-squares estimation yields a bandwidth of 0.236 and 0.445 for the UK and New Zealand, respectively. The Ljung-Box statistics indicate the absence of serial correlation in the estimated residuals in both cases. Our estimates of the bandwidth indicate that the inactive band in New Zealand is wider than that in the UK. This can be explained by the fact that the distance between the United States and the UK is shorter than that between the United States and New Zealand, implying lower transaction costs and, hence, a narrow band for the UK.

The likelihood ratio statistic (LR1) for the unit-root hypothesis in the middle regime, $H_0^{C(1)}$, fails to reject the hypothesis of the unit-root property within the band, since the corresponding p values for the UK and New Zealand are 0.112 and 0.418, respectively. Our main assertion is that while real exchange rates exhibit the unit-root property within a nonarbitrage band, they reveal the tendency of mean reverting when deviations are profitably large. Since we have found that the unit-root hypothesis in the middle regime is not rejected, we then examine whether real exchange rates follow an I(1) process in all three regimes, $H_0^{C(2)}$. The likelihood ratio statistic (LR2) for the hypothesis, $H_0^{C(2)}$, is 13.517 and 12.310 for the UK and New Zealand, respectively, which rejects the unit-root hypothesis of real exchange rates in the middle and outer regimes at the 10% level of significance. By combining the previous results with the result that real exchange rates follow an I(1) process within the band, we conclude that large deviations from PPP appear to be mean reverting, while small ones do not.

After finding that the TPI-based real exchange rate is stationary with a nonlinear adjustment, we then attempt to determine whether the mean-reverting adjustment of the real exchange rate is mainly attributable to the adjustment of prices or nominal exchange rates, or both. For this purpose, we estimate a multivariate threshold vector-error-correction model and report our results in Table 3.

The lag length for the TVECM is selected as follows. We first determine the lag order based on the multivariate AIC, as proposed by Paulsen (1984). The whiteness of the estimated residuals

Table 3. Results for the Threshold Vector-Error-Correction Model

	λ_1^s	λ_1^p	λ_1^{p*}
UK			
L and U	−0.004 (0.08)	0.07* (0.03)	−0.003 (0.02)
	$Q^s(16) = 22.57$ [0.13]	$Q^P(16) = 12.78$ [0.69]	$Q^{p^*}(16) = 15.46$ [0.49]
	$Q^{2,s}(16) = 20.26$ [0.21]	$Q^{2,p}(16) = 22.52$ [0.13]	$Q^{2,p^*}(16) = 5.66$ [0.99]
NZ			
L and U	−0.29* (0.11)	0.06 (0.07)	−0.04 (0.03)
	$Q^s(16) = 12.98$ [0.67]	$Q^P(16) = 16.92$ [0.39]	$Q^{p^*}(16) = 9.91$ [0.87]
	$Q^{2,s}(16) = 6.43$ [0.98]	$Q^{2,p}(16) = 13.34$ [0.65]	$Q^{2,P^*}(16) = 16.62$ [0.41]

UK and NZ indicate the United Kingdom and New Zealand, respectively. L and U represent the regimes that are defined as $q_{t-1} \leq -\tau$, and $q_{t-1} \geq \tau$, respectively. Terms λ_1^s, λ_1^P, and λ_1^{p*} denote the error-correction coefficients in Equation 2. The numbers in parentheses under the estimates are the standard deviations of the respective estimates. The numbers in brackets are p values. $Q^s(j)$, $Q^p(j)$, and $Q^{p^*}(j)$ are the Ljung-Box autocorrelation test statistics for up to jth-order autocorrelation for the estimated residuals from the equations of Δs_t, Δp_t, and Δp_t^*, respectively. $Q^{2,s}(j)$, $Q^{2,p}(j)$, and $Q^{2,P^*}(j)$ are the test statistics for up to jth-order autocorrelation for the estimated squared residuals from the equations of Δs_t, Δp_t, and Δp_t^*, respectively, which are χ^2 distributions with j degrees of freedom.
* Significance at the 5% level.

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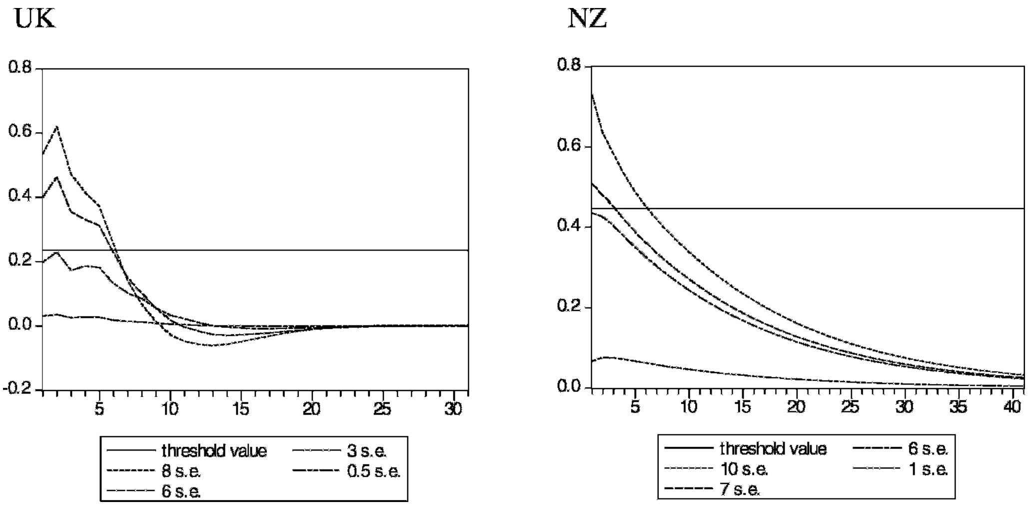

Figure 2. GIRF for the UK and New Zealand under Different Magnitudes of Shock

is then examined for each equation using the Q statistic. If the residuals in any equation prove to be nonwhite, we then sequentially choose a higher lag structure until they are whitened. The lag length for the TVECM is therefore set to be 3 for both the UK and New Zealand.

Table 3 reports the estimates of error-correction coefficients λ_1^s, λ_1^p, and λ_1^{p*}, which govern the adjustment to PPP. For the UK, the error-correction coefficient in the UK price equation is 0.07 and is significant at the 5% level, which implies that it is primarily the UK price that adjusts to restore long-run equilibrium when deviations from PPP occur. This finding is consistent with the prediction of Dornbusch (1976). For New Zealand, the error-correction coefficient in the exchange rate equation is -0.29 and is significant at the 5% level. In other words, the mean-reverting adjustment of the real New Zealand dollar rate is attributed to the adjustment of the nominal New Zealand dollar rate, which is consistent with that described in Cheung, Lai, and Bergman (2004). In addition, both the Q and Q^2 statistics reveal that there is neither serial correlation nor autoregressive conditional heteroskedasticity in the residuals at the 5% level of significance.

Based on a linear framework, Cheung, Lai, and Bergman (2004) show that nominal exchange rate adjustment is the key engine governing the speed of PPP convergence in five major industrial countries. Our findings indicate that the sources of the real exchange rate adjustments toward PPP are country dependent in the nonlinear framework. Our findings in Table 3 are not affected if a two-variable, $(s_t, p_t^* - p_t)$, TVECM is applied.[9]

Given the fact that our model supports the nonlinear band-TAR specification of real exchange rates, we are interested in whether the half-life of real exchange rates estimated from a band-TAR model is consistent with the sticky price explanation. For this purpose, we calculate the half-life of real exchange rates using generalized impulse response functions. The plots of the GIRFs are given in Figure 2 for the UK and New Zealand, respectively, in which the GIRFs are nonmonotonic and shock dependent. The GIRF is highly persistent when the sizes of the shocks

[9] With a two-variable TVECM, we find that the mean-reverting adjustment of the real pound rate and the real New Zealand dollar rate is mainly attributed to the adjustment in the price differential between the United States and the UK (p^*-p) and the nominal New Zealand dollar rate, respectively. The empirical results are not reported here but are available upon request from the authors.

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Table 4. Estimated Half-Lives (in quarters)

	UK			NZ		
	Sizes of the Shock (SE)	h_1	h_2	Sizes of the Shock (SE)	h_1	h_2
Within the band	0.5	—	—	1	—	—
	3	—	—	6	—	—
Outside of the band	6	3	4	7	2	2
	8	3	4	10	2	2

UK and NZ indicate United Kingdom and New Zealand, respectively. h_1 represents the half-lives calculated from the peak of GIRF to the band edge of the equilibrium, and h_2 represents the half-lives calculated from the beginning of GIRF to the band edge of the equilibrium. SE indicates standard error of residuals. The '—' symbol indicates that the half-life is not calculated.

are small, such that the initial response of the real exchange rate is smaller than the band. In general, we find that the larger the initial deviation, the shorter the estimated half-life.

It is worth noting that in a band-TAR model, the entire interval $[-\kappa, \kappa]$ of the band is the equilibrium of real exchange rate, in which there is no arbitrage. Therefore, the half-life should be constructed relative to the band edge rather than the center of the equilibrium. The half-life in our article is measured by how long it takes from the GIRF's peak (initial) to dissipate by half relative to the band edge of the equilibrium, which we indicate by h_1 (h_2). Table 4 displays the estimated half-lives of various sizes of shock to the real exchange rates based on the GIRF. Since the half-life is calculated relative to the band edge, we consider the cases with large shocks, in which the initial response to the shocks is greater than the band. In the cases of the UK and New Zealand, the estimated half-lives for both h_1 and h_2 are about two to four quarters with a large shock. These results shed light on Rogoff's (1996) PPP puzzle.

4. Conclusion

The purpose of this article was to address several interesting issues regarding PPP. We find that the mean-reverting adjustment toward PPP is nonlinear and sensitive to price indices. The nonlinear mean-reverting adjustment of the real exchange rates is detected when TPI-based rates are applied. Based on the TPI-based rate, we find that the adjustment toward PPP is mainly attributed to the adjustment in the price level of the UK. As for New Zealand, the adjustment is attributed to the nominal New Zealand dollar rate. Finally, we point out that the half life should be constructed relative to the band edge in a band-TAR model. The estimated half-life of the TPI-based rate with a large shock is shorter than conventional estimates and is consistent with the sticky price explanation.

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